

5-9 Complex Numbers

$$\sqrt{-1} = i$$

$$i^2 = -1$$

Simplify.

$$\begin{array}{llll} \sqrt{-5} & \sqrt{-25} & \sqrt{-50} & \sqrt{-28} \\ i\sqrt{5} & 5i & i\sqrt{50} & 2i\sqrt{7} \\ & & 5i\sqrt{2} & \end{array}$$

$$\begin{array}{l} i^{17} \\ \cancel{i^2} \\ i \cdot i^{16} \\ i (i^2)^8 \\ i (-1)^8 \\ \boxed{i} \end{array}$$

$$\begin{array}{l} i^{46} \\ (i^2)^{23} \\ (-1)^{23} \\ \boxed{-1} \end{array}$$

Multiplication

ex:

$$\begin{array}{l} -3i \cdot 2i \\ -6i^2 \\ \underbrace{(-)}_{(-)} \\ \boxed{6} \end{array}$$

Must simplify 1st!

ex:

$$\begin{aligned} &\sqrt{-12} \cdot \sqrt{-2} \\ &i\sqrt{12} \cdot i\sqrt{2} \\ &\overset{i^2}{\downarrow} \sqrt{24} \\ &\quad \quad \quad \text{"6"} \\ &-2\sqrt{6} \end{aligned}$$

ex:

$$\begin{aligned} &\sqrt{-4} \cdot \sqrt{-25} \\ &2i \cdot 5i \\ &\quad \quad 10i^2 \\ &\quad \quad -10 \end{aligned}$$

ex:

$$\begin{aligned} &\sqrt{-3} \cdot \sqrt{-27} \\ &i\sqrt{3} \cdot 3i\sqrt{3} \\ &\quad \quad 3i^2 \cdot 3 \\ &\quad \quad -9 \end{aligned}$$

Add and Subtract

ex:

$$\begin{aligned} &\sqrt{-16} - \sqrt{-49} \\ &4i - 7i \\ &\quad \quad \boxed{-3i} \end{aligned}$$

ex:

$$i\sqrt{2} + 3i\sqrt{2}$$

$$4i\sqrt{2}$$

Solve.

$$5y^2 + 20 = 0$$

$$5y^2 = -20$$

$$\sqrt{y^2} = \sqrt{-4}$$

$$y = \pm 2i$$

Cannot leave i in the denominator

$$\frac{2}{3i} \cdot \frac{i}{i} = -\frac{2i}{3}$$

\downarrow
 $3i^2$
 \downarrow
 -3

ex:

$$\frac{6}{\sqrt{-2}} = \frac{6}{i\sqrt{2}} \cdot \frac{i\sqrt{2}}{i\sqrt{2}} = \frac{6i\sqrt{2}}{-2}$$

$$= -3i\sqrt{2}$$

Complex Numbers

 $a + bi$ where $a, b \in \mathbb{R}$ ex: $3+2i$; $6-4i$

Pure imaginary number

 bi where $b \neq 0$ ex: $2i$; $i\sqrt{3}$

ex:

$$(3 + 6i) + (4 - 2i)$$

$$7 + 4i$$

ex:

$$(3 + 6i) - (4 - 2i)$$

$$-1 + 8i$$

ex:

$$(3 + 4i)(5 + 2i)$$

$$15 + 6i + 20i + 8i^2$$

$$7 + 26i$$

ex:

$$(8 + 3i)^2$$

ex:

$$\frac{(4 - 3i)(2 + 4i)}{(2 - 4i)(2 + 4i)}$$

$$\frac{8 + 10i + 12i^2}{4 - 16i^2} = \frac{20 + 10i}{20} = \frac{2 + i}{2}$$

"Fun with Factoring"

$$x^2 + 8$$

Hw

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21-41 odd 44, 50, 56